Seat No. _______________

H-003-1016001

B. Sc. (Sem. VI) Examination

April - 2023

Maths : Paper - 08

(Graph Theory & Complex Analysis - II) (Old Course)

Faculty Code : 003
Subject Code : 1016001

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) Attempt all questions.
(2) Right hand side figures indicate the marks.

1 (a) Answer the following questions in short : **4**

(1) Define empty graph.

(2) Define isolated vertex.

(3) State the graph theory's second theorem.

(4) Define circuit.

(b) Attempt any **one** : **2**

(1) State and prove first theorem of graph theory.

(2) Prove the complete graph K_n has always a Hamiltonian circuit.

(c) Attempt any **one** : **3**

(1) Obtain the numbr of pendant vertices in a binary tree with n vertices.

(2) If any graph has exatly two vertices of odd degrree then there must be a path joining these two vertices.

(d) Attempt any **one** : **5**

(1) Prove a tree with n vertices has $n-1$ edges.

(2) State and prove necessary and sufficient condition for a graph to become a disconnected graph.

2 (a) Answer the following questions in short : **4**

(1) Define planer graph.

(2) Define separable graph.

(3) Define circuit vector.

(4) Define path matrix.

(b) Attempt any **one** : **2**

(1) Define incidence matrix with example.

(2) In any simple connected planner graph wtih f region, n vertex, e edges $(e > 2)$ prove $e \leq 3n - 6$.

(c) Attempt any **one** : **3**

(1) Prove $(W_T, \oplus, \cdot)$ is a subgroup of W_G over the field GF_2.

(2) Every tree with two or more vertices is 2-cromatic.

(d) Attempt any **one** : **5**

(1) Prove complete graph K_5 is non-planar.

(2) If G is a graph with n vertex, e edges, f faces, k components then prove $n - e + f = k + 1$.

3 (a) Answer the following questions in short : **4**

(1) Define inversion mapping.

(2) Define cross ratio of bilinear map.

(3) Find fixed points of $W = \dfrac{6z - 9}{z}$.

(4) Define fixed point.

(b) Attempt any **one** : **2**

(1) Discuss fixed points of bilinear map.

(2) Define conformal mapping.

(c) Attempt any **one** : **3**

(1) Find the mobius mapping which transforms points $Z = -1, \infty, 1$ of Z plane into $W = 2, 1, 0$ of W plane.

(2) Prove the transformation $W = \dfrac{1+Z}{1-Z}$ maps the region $|Z| \leq 1$ of Z plane into $\operatorname{Re}(W) \geq 0$.

(d) Attempt any **one** : **5**

(1) Prove the composition of two bilinear map is again a bilinear map.

(2) Obtain the image of circle $|Z-3|=5$ of Z plane under the transformation $W=\frac{1}{Z}$.

4 (a) Answer the following questions in short : **4**

(1) Define power series.

(2) Write the maclourin's series for $\cosh z$.

(3) Find the radius of convergence for $\sum_{1}^{\infty}\frac{z^n}{g^n+1}$.

(4) Write the maclourin's series for $\sin z$.

(b) Attempt any **one** : **2**

(1) Prove $\frac{\sinh z}{z^2}+\frac{1}{z}+\sum_{n=1}^{\infty}\frac{z^{2n-1}}{(2n+1)!}$.

(2) If series $\sum z_n$ is absolute convergent then $\sum z_n$ is also convergent.

(c) Attempt any **one** : **3**

(1) Prove $\frac{1}{z^2\sinh z}=\frac{1}{z^3}-\frac{1}{6z}+\frac{7}{360}z\,......$

(2) Prove $\frac{1}{4z-z^2}=\sum_{n=0}^{\infty}\frac{z^{n-1}}{4^{n+1}}$ where $0<|z|<4$.

(d) Attempt any **one** : **5**

(1) State and prove Taylor's series for an analytic function.

(2) In usual notation prove that

$$\cosh\left(z+z^{-1}\right)=\sum_{n=-\infty}^{\infty}a_n z^n$$

5 (a) Answer the following questions in short : **4**

(1) Define singular point.

(2) Write the principle part of Laurent's expansion.

(3) Write the formula for finding residue of $f(z)$ at simple pole z_0.

(4) Find residue of $f(z)=\dfrac{z+2}{(z-1)(z-2)}$ at $z_0=1$.

(b) Attempt any **one** : **2**

(1) Evaluate $\int_C \dfrac{(5z-2)}{z(z-1)}dz$ where C; $|z|=\dfrac{1}{2}$.

(2) Evaluate $\int_C \dfrac{(2z+3)}{z(z-1)}dz$ where C; $|z|=\dfrac{1}{4}$.

(c) Attempt any **one** : **3**

(1) If z_0 is the m^{th} order pole of $f(z)$ then

$$\text{Res}\left(f(z), z_0\right)=\frac{\phi^{(m-1)}(z_0)}{(m-1)!}.$$

(2) Evaluate $\int_0^\infty \dfrac{x^2dx}{(x^2+g)(x^2+4)}$.

(d) Attempt any **one** : **5**

(1) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{5+4\sin\theta}$.

(2) State and prove Cauchy-Residue Theorem.
